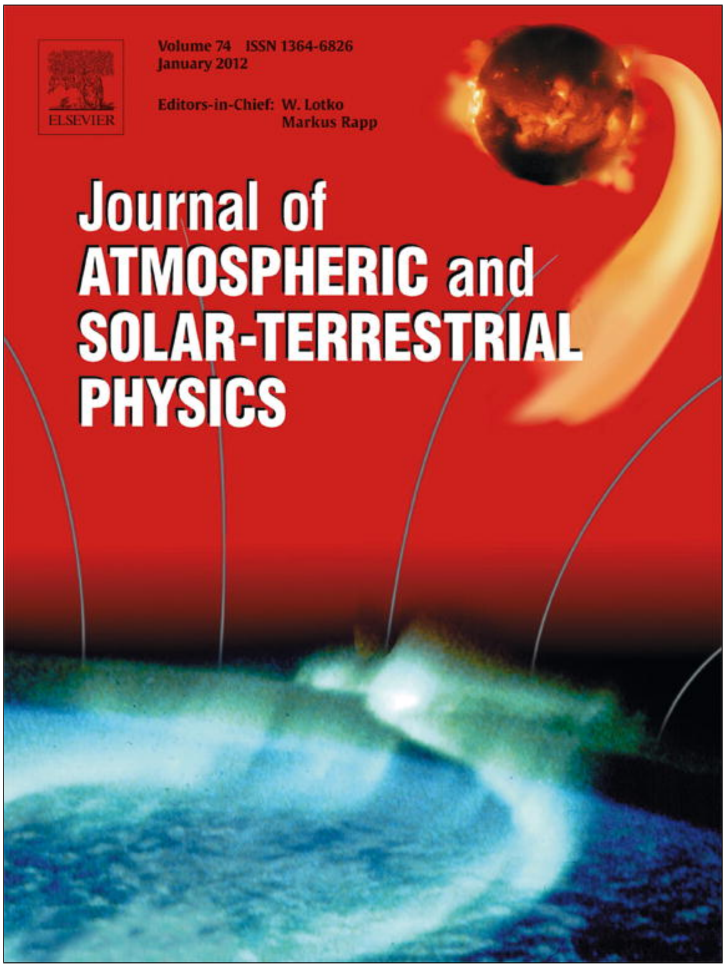

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Does the Sun work as a nuclear fusion amplifier of planetary tidal forcing? A proposal for a physical mechanism based on the mass-luminosity relation

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ABSTRACT

Numerous empirical evidences suggest that planetary tides may influence solar activity. In particular, it has been shown that: (1) the well-known 11-year Schwabe sunspot number cycle is constrained between the spring tidal period of Jupiter and Saturn, ~ 9.93 year, and the tidal orbital period of Jupiter, ~ 11.86 year, and a model based on these cycles can reconstruct solar dynamics at multiple time scales (Scafetta, in press); (2) a measure of the alignment of Venus, Earth and Jupiter reveals quasi 11.07-year cycles that are well correlated to the 11-year Schwabe solar cycles; and (3) there exists a 11.08 year cyclical recurrence in the solar jerk-shock vector, which is induced mostly by Mercury and Venus. However, Newtonian classical physics has failed to explain the phenomenon. Only by means of a significant nuclear fusion amplification of the tidal gravitational potential energy dissipated in the Sun, may planetary tides produce irradiance output oscillations with a sufficient magnitude to influence solar dynamo processes. Here we explain how a first order magnification factor can be roughly calculated using an adaptation of the well-known *mass-luminosity relation* for main-sequence stars similar to the Sun. This strategy yields a conversion factor between the solar luminosity and the potential gravitational power associated to the mass lost by nuclear fusion: the average estimated amplification factor is $A \approx 4.25 \times 10^6$. We use this magnification factor to evaluate the theoretical luminosity oscillations that planetary tides may potentially stimulate inside the solar core by making its nuclear fusion rate oscillate. By converting the power related to this energy into solar irradiance units at 1 AU we find that the tidal oscillations may be able to theoretically induce an oscillating luminosity increase from 0.05–0.65 W/m^2 to 0.25–1.63 W/m^2, which is a range compatible with the ACRIM satellite observed total solar irradiance fluctuations. In conclusion, the Sun, by means of its nuclear active core, may be working as a great amplifier of the small planetary tidal energy dissipated in it. The amplified signal should be sufficiently energetic to synchronize solar dynamics with the planetary frequencies and activate internal resonance mechanisms, which then generate and interfere with the solar dynamo cycle to shape solar dynamics, as further explained in Scafetta (in press). A section is devoted to explain how the traditional objections to the planetary theory of solar variation can be rebutted.

1. Introduction

It is currently believed that solar activity is driven by internal solar dynamics alone. In particular, the observed quasi-periodic 11-year sunspot and total solar irradiance (TSI) cycles are believed to be the result of solar differential rotation, which is modeled in hydromagnetic solar dynamo models (Tobias, 2002). However, the severe incompleteness of the current solar theories assuming that the Sun acts as an *isolated system* is indirectly demonstrated by their inability to reconstruct solar variability occurring at multiple time scales, as also critics of the planetary theory acknowledge (de Jager and Versteegh, 2005). On the contrary, since the 19th century (Wolf, 1859) a theory has been proposed claiming that solar dynamics is partially driven by planetary tides. This theory has never been definitely disproved, although a plausible physical mechanism has not been discovered yet.

Indeed, a planetary theory of solar variation has been criticized for various reasons (Charbonneau, 2002). There are three classical objections. A first objection claims that planetary tides alone would not explain solar variability (Smythe and Eddy, 1977). A second objection based on classical physics claims that the planetary tidal forces are too small to modulate solar activity: for example, tidal accelerations at the tachocline level are about 1000 times smaller than the accelerations of the convective motions (de Jager and Versteegh, 2005; Callebaut et al., 2012).

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A third objection claims that traditional concepts in the theory of stellar structure such the Kelvin–Helmholtz time scale (Mitalas and Sills, 1992; Stix, 2003) would predict that, because the erratic propagation of the light from the core to the convective zone requires 10^4–10^8 years, changes in the luminosity production would be smoothed out before reaching the convective zone, and the smoothed anomaly signal would not be observable in the final luminosity output. The above three major objections have prevented solar scientists from further investigating the issue taking advantage of the larger and more accurate satellite data about solar activity collected since 1980s, of computer modeling and of theoretical thinking based on modern physics.

On the contrary, the argument that solar activity may be linked to planetary motion would be supported by a large number of empirical evidences (Wolf, 1859; Schuster, 1911; Takahashi, 1968; Bigg, 1967; Jose, 1965; Wood and Wood, 1965; Wood, 1972; Dingle et al., 1973; Okal and Anderson, 1975; Fairbridge and Shirley, 1987; Charvátová and Střeštík, 1991; Charvátová, 2000, 2009; Landscheidt, 1988, 1999; Juckett, 2003; Hung, 2007; Wilson et al., 2008; Scafetta, 2010; Perryman and Schulze-Hartung, 2010; Scafetta, 2012). Recently, Scafetta (in press) has shown that it is possible to reconstruct solar variability with a very good accuracy at the decadal, secular and millennial scale throughout the Holocene using a model based on the tidal cycles of Jupiter and Saturn plus the dynamo cycle. Scafetta's results have strongly rebutted the empirical criticism by Smythe and Eddy (1977), for example, and have reopened the issue.

The geometrical patterns of the motion of the Sun relative to the barycenter of the solar system have been used by some of the above authors to support a planetary modulation of solar activity. The complex wobbling of a star around the barycenter of its solar system is a well-known phenomenon of stellar motion (Perryman and Schulze-Hartung, 2010). Indeed, Wolff and Patrone (2010) have recently proposed that the rotation of the Sun around the barycenter of the solar system could induce small mass exchanges that release potential energy. The mass exchange would also carry fresh fuel to deeper levels and increase solar activity. This phenomenon would cause stars like the Sun with an appropriate planetary system to burn somewhat more brightly and have shorter lifetimes than identical stars without planets. However, the solar barycentric motion should be understood just as an approximate geometrical proxy of the forces acting on the Sun. Tidal forces, torques and jerk shocks act on and inside the Sun, which is not just a point-size body in free fall. Only these forces can potentially influence solar activity according to the laws of mechanics, although additional more complex mechanisms cannot be excluded.

Herein, we observe that the continuous tidal *massaging* of the Sun should be heating the solar core too. Tidal heating, where orbital and rotational energy is dissipated as heat through internal friction processes, is a well-known planetary phenomenon (Goldreich and Soter, 1966; Jackson et al., 2008). Particularly, tidal heating effects are macroscopic in the case of Jupiter's moon Io (Bennett et al., 2010). In the case of a star, we hypothesize that tidal heating modulaes the nuclear fusion rate and, therefore, the star's total solar irradiance (TSI) output. Thus, the planetary gravitational tidal energy dissipated in the core should be greatly amplified by the internal nuclear fusion rate response to it. We propose a rough estimate of both the value of the planetary gravitational tidal heating of the solar core and of the internal nuclear feedback amplification factor that would amplify the released gravitational tidal power into TSI output. Finally, we compare the magnitude of the planetary tidal induced TSI output variation against the observed TSI variation. The theoretical results of this paper, which are based on modern physics, would rebut the second major objection against the planetary-solar theory, which uses arguments based on classical physics alone to claim that planetary tides are too small to influence the Sun (de Jager and Versteegh, 2005). Indeed, the failure of the 19th century Kelvin-Helmholtz timescale theory claiming that the Sun is about 10 million years old instead of the currently accepted age of 4.7 billion years (Carroll and Ostlie, 2007) demonstrates that classical physics alone does not explain how the Sun works by a large factor. Indeed, the well-known fact that stars are not classical physical systems can invalidate any argument that uses classical physics alone to disprove a planetary tidal influence on the Sun.

The third objection is not explicitly addressed herein. However, the interior of the Sun is not in a pure hydrostatic equilibrium, but it is likely crossed by buoyancy gravitation-waves known as g-mode waves (García et al., 2007). Wolff and Patrone (2010) proposed that g-mode waves may activate extremely fast upward transport mechanisms of luminosity variation from the core to the surface. Fast wave propagation mechanisms would solve the theoretical problems related to the theorized extremely slow luminosity diffusion movement occurring in the radiative zone (Mitalas and Sills, 1992; Stix, 2003).

In Section 2 we summarize some empirical findings suggesting that planetary tides may influence solar activity. In Section 3 we develop a simple energetic model argument to demonstrate that such a theory may be physically plausible. We conclude that the empirical and theoretical evidences in favor of a planetary influence on solar activity are too strong to be ignored: in the future there is the need to study them extensively and include these mechanisms in future solar models. This would be greatly beneficial to solar physics and climate science as well.

2. Empirical evidences for a planetary forcing on the Sun

The possibility that solar cycles are partially modulated by planetary tidal cycles has been frequently suggested since the 19th century (Wolf, 1859; Brown, 1900; Bendandi, 1931; Schuster, 1911). For example, in a short letter to Carrington, Wolf (1859) proposed that the variations of sunspot-frequency depend on the influences of Venus, Earth, Jupiter and Saturn. Brown (1900) noted that the 11-year sunspot cycle and its multidecadal variation could be linked to the combined influence of Jupiter and Saturn, which should produce tidal-rising cycles with periods of about 10, 12 and 60 years. Jose (1965) noted that the barycentric motion of the Sun presents a 178.7 year periodicity and that the rate of change of the Sun's orbital angular momentum can be correlated to both the 11-year sunspot cycle and the 22-year magnetic dipole inversion and sunspot polarity Hale cycle. Bigg (1967) found that the daily sunspot number for the years 1850–1960 presents a consistent periodicity at the sidereal period of Mercury, which is partially modulated by the position of Venus, Earth and Jupiter. In a set of studies it has been noted a link between aurora occurrences and planetary cycles (Charvátová et al., 1988; Scafetta, 2012) and a coherence between solar motion and solar variability at multiple frequencies ranging from a 60-year periodicity, which is related to the great conjunction period of Jupiter and Saturn, to a 2400 year (Charvátová and Střeštík, 1991; Charvátová, 2000; Scafetta, 2010). Landscheidt (1988, 1999) noted that solar rotation and extremes in sunspot cycle are correlated to solar motion and that this correlation had very small probabilities, ~ 0.001, of being accidental. Further evidence of a link involving spin–orbit coupling between the Sun and the Jovian planets has been recently proposed (Juckett, 2003; Wilson et al., 2008). Ogurtsov et al. (2002) and Komitov (2009) showed that millennial sunspot and cosmogenic isotope records such as ^{14}C and ^{10}Be are characterized by major cycles such as at about 45, 60, 85, 128, and 205 years. These cycles can be easily associated to some combination of planetary

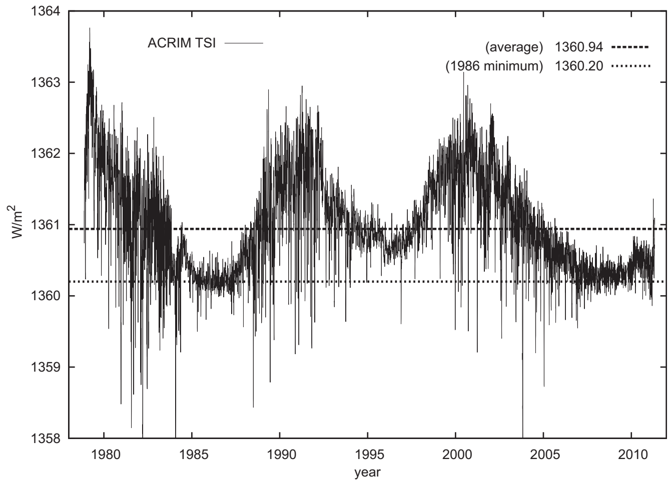

Fig. 1. ACRIM total solar irradiance (TSI) satellite composite at 1 AU. The average value is 1360.94 W/m^2, which implies an average solar luminosity of $L_S = 3.827 \times 10^{26}$ W. (Data from http://acrim.com.)

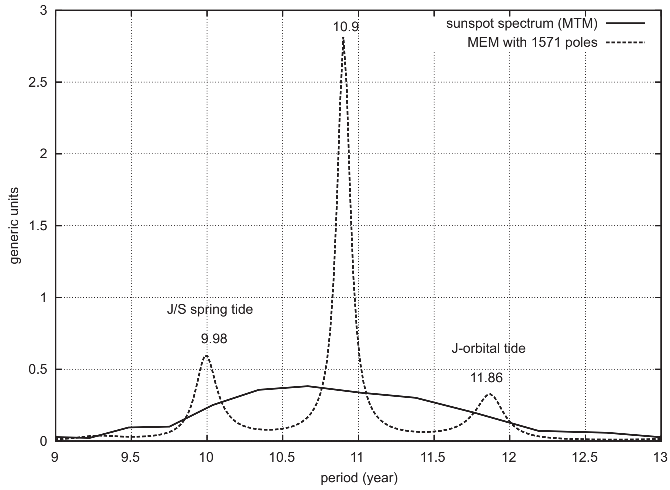

Fig. 2. Power spectrum analysis of the 11-year Schwabe's solar cycle as deduced from the sunspot number record monthly sampled from 01/1749 to 12/2010 (3144 data) (http://sidc.oma.be/sunspot-data/). Multi-taper method (MTM) and maximum entropy method (MEM) with 1572 pole order are used (Ghil et al., 2002). In the latter case a reasonable error of about ± 0.1 year related to the monthly sampling can be assumed. The three peaks have a 99% confidence level against red noise background. See Scafetta (in press) for details.

cycles. For example: $\sim$45-year is the synodic period of Jupiter and Uranus; $\sim$60 year is the great conjunction cycle of Jupiter and Saturn (which is made of three J/S conjunction periods); $\sim$85-year is the 1/7 resonance of Jupiter and Uranus; and $\sim$205 year is the beat resonance between the 60-year and the 85-year cycles.

Twenty-five of the 38 largest known solar flares were observed to start when one or more tide-producing planets (Mercury, Venus, Earth, and Jupiter) were either nearly above the event positions ($<10^\circ$ longitude) or at the opposing side of the Sun (Hung, 2007). Active solar regions are almost symmetrical with respect to the solar equator and are limited to the medium and low latitudes probably because of the influence of the planetary tidal force that are stronger at the medium and low latitudes (Takahashi, 1974). Actual TSI satellite observations (Scafetta and Willson, 2009), in particular during the last solar maximum 1999–2004, present very large quasi-annual and sub-annual cycles in phase with inner planet orbits: for example, Fig. 1 shows the ACRIM TSI composite where these large annual cycles from 1999 to 2004 are macroscopic. These annual cycles may emerge in solar records only during specific periods as the result of some resonance effect.

A detailed study of the cycles of the Earth's climate and of the cycles of the speed of the barycentric motion of the Sun has found that the two systems are strongly coherent at multiple frequencies ranging from 5 to 100 years (Scafetta, 2010, 2012). In particular, cycles at about 10.4, 20, 30 and 60 years, which are mostly related to Jupiter and Saturn orbits, are clearly seen in both climate and solar barycentric motion. This finding would suggest that the planets are driving solar variability that then drives the Earth's climate. Note that the 10.4 year cycle seen in the Earth's temperature record is related to a multiple-planetary alignment, as also found by researchers that have critiqued the possibility of a planetary-tidal theory (Okal and Anderson, 1975). This cycle is likely dominated by the $\sim$10-year spring tidal period of Jupiter and Saturn, which is half of their $\sim$20-year synodic period.

In the following subsections we show that the three subsystems Jupiter–Saturn, Venus–Earth–Jupiter and Mercury–Venus produce major resonances that are centered on the 11-year Schwabe solar cycle. These results suggest that it is unlikely that the Sun is oscillating around the Schwabe's frequency band (9–13 year periods) just by coincidence. Empirical evidences clearly suggest that solar dynamics is approximately synchronized to planetary motion.

2.1. The 10–12 year Jupiter and Saturn cycles in the sunspot number record

Scafetta (in press) has recently shown that the sunspot number record presents a wide 9–13 year Schwabe's frequency band made of three frequencies. These are compatible with Jupiter/Saturn spring tidal period of 9.93 years (note that ephemeris calculations of the orbits of Jupiter and Saturn since 1749 give a J/S spring tidal cycle of 9.93 ± 0.5 year), with a central 10.87 ± 0.1 year period, which is likely related to a central 11-year solar dynamo cycle, and with the period of Jupiter at 11.86 years. These patterns suggest that the 11-year solar cycle may be partially induced by some solar resonance driving the solar dynamo that amplifies the beat average cycle (10.81–10.90 years) induced by the two major tides produced by the combined system of Jupiter–Saturn ($\sim$9.93-year cycle) and by Jupiter's orbit ($\sim$11.86-year cycle), plus a possible small dynamical correction induced by other tides as explained below. Fig. 2 shows the three frequency peaks of the sunspot number record. According to the power spectrum evaluation, it may be possible to associate at least 0.1–0.5 W/m^2 variability to the planetary tidal effect.

The existence of three cycles imply additional beat cycles. For example, there exists a beat at

$$T = \left(\frac{1}{T_{JS}} - \frac{1}{T_J}\right)^{-1} = 61 \pm 2 \text{ year}. \quad (1)$$

This quasi-sexagesimal cycle is found in millennial cosmogenic isotope records such as ^{14}C and ^{10}Be and in the aurora records (Charvátová et al., 1988; Ogurtsov et al., 2002; Komm et al., 2003), and in numerous terrestrial climate records (Scafetta, 2010, in press). Other beats occur at about 115, 130 and 983 years. Scafetta (in press) extensively studied the above findings and showed that these frequencies can be used to explain all known major solar patterns, which include a sufficiently correct timing of the 11-year Schwabe solar cycle and of the known grand solar minima during the last millennium known as the Oort, Wolf, Spörer, Maunder and Dalton minima, plus the emergence and the timing of observed quasi-millennial solar and climate cycles for 12,000 years.

2.2. The Venus–Earth–Jupiter 11.07-year alignment cycle

Venus, Earth and Jupiter are the three major tidal planets (see Section 3 and Table 2 for the estimated values of the tidal elongation induced on the Sun by each planet). It has been found that Venus, Earth and Jupiter tend to be mostly aligned every 11 years (Bendandi, 1931; Takahashi, 1968; Wood, 1972; Hung, 2007). Indeed, when the combined alignment of Venus, Earth and Jupiter is taken into account, the configuration E–V–Sun–J or Sun–V–E–J repeats with a period of approximately 22 years, and the correspondent tidal period is half of it. This period can be calculated using the following resonance formula:

$$P_{VEJ} = \frac{1}{2}\left(\frac{3}{P_V} - \frac{5}{P_E} + \frac{2}{P_J}\right)^{-1} = 4043\ \text{day} = 11.07\ \text{year}, \quad (2)$$

where P_V=224.701 days, P_E=365.256 days and P_J=4332.589 days are the sidereal periods of Venus, Earth and Jupiter, respectively.

It is possible to define a very simple multi-planetary alignment index for Venus, Earth and Jupiter (Hung, 2007). Let us assume that the Sun is in the center of a system and that three planets are orbiting the Sun. An alignment index I_{ij} between planet i and planet j can be defined as

$$I_{ij} = |\cos(\Theta_{ij})|, \quad (3)$$

where Θ_{ij} is the angle between the positions of the two planets relative to the solar center. Eq. (3) indicates that when the two planets are aligned ($\Theta_{ij} = 0$ or $\Theta_{ij} = \pi$) the alignment index gives the largest value because these two positions imply the largest combined tidal elongation on the Sun, which is known as the *spring tide*. This is the synodic/opposition tidal period (which is half of the synodic period of the two planets because tidal forces produce two opposite tides). When $\Theta_{ij} = \pi/2$ the index gives the lowest value because at right angles the tides of the two planets tend to cancel each other, which is known as the *neap tide*. In the case of a system made of Venus, Earth and Jupiter, there are three correspondent alignment indexes

$$I_V = |\cos(\Theta_{VE})| + |\cos(\Theta_{VJ})|, \quad (4)$$

$$I_E = |\cos(\Theta_{EV})| + |\cos(\Theta_{EJ})|, \quad (5)$$

$$I_J = |\cos(\Theta_{JV})| + |\cos(\Theta_{JE})|. \quad (6)$$

Finally, we can define a combined alignment index I_{VEJ} for the three planets as

$$I_{VEJ} = \text{smallest among } (I_V, I_E, I_J). \quad (7)$$

The above index I_{VEJ} has a value ranging from 0 to 2. It can be carefully calculated using the daily coordinates of Venus, Earth and Jupiter relative to the Sun, which can be obtained by using the high-precision JPL HORIZONS ephemerides system (http://ssd.jpl.nasa.gov/).

Fig. 3 shows two alternative comparisons between the annual average of sunspot number since 1800 and two alternative indexes obtained with the alignment index I_{VEJ}. In the top of the figure the sunspot number record is compared against the number of days (every four years) that present an alignment index $I_{VEJ} > 66\%$; in the bottom of the figure the same sunspot number record is compared against the most aligned days with $I_{VEJ} > 95\%$ (or $I_{VEJ} > 1.9$).

The figure shows that there exists a very good coherence between the sunspot number record and the two indexes obtained by using the alignment index I_{VEJ}. In particular, note the relatively good phase synchronization. The two records present 19 full cycles for the covered period 1800–2010 with an average period of about 11.05 year per cycle, which is compatible with Eq. (2). Note that the average of the observed 23 sunspot cycle lengths since 1750 is about 11.06 years.

2.3. The 11.08-year solar jerk-shock vector cycle

The jerk vector is the time derivative of the acceleration vector (Schot, 1978) and it is commonly used in engineering as a measure of the shocks and stresses felt by an object subject to a varying acceleration. Jerk functions are associated with dissipative chaotic flow mechanisms (Sprott, 1997), and changes in the acceleration of vortices are associated with the generation of flow noises (Schot, 1978). Jerk stresses may contribute to solar variations (Wood and Wood, 1965). The Sun's acceleration vector is given by

$$\vec{a}_S(t) = \sum_{P=1}^{8} \frac{Gm_P \vec{R}_{SP}(t)}{R_{SP}^3(t)}, \quad (8)$$

where $\vec{R}_{SP}(t)$ is the vector position of a planet from the Sun. The sum is extended to the eight planets of the solar system (Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus and Neptune).

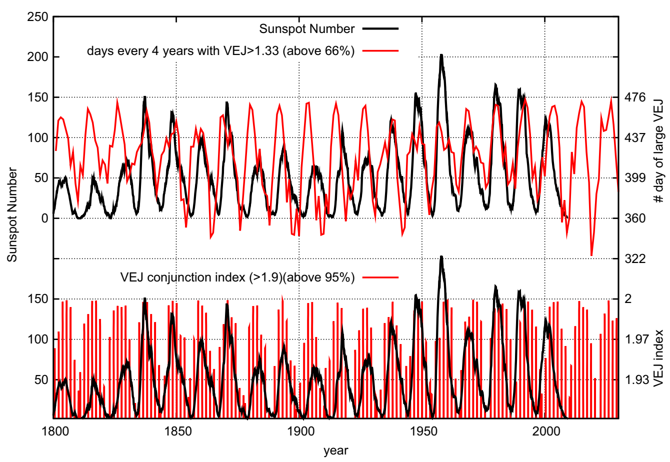

Fig. 3. (Top) the sunspot number record (black) is compared against the number of days (every four years) (red/gray) when the alignment index $I_{VEJ} > 66\%$. (Bottom) the sunspot number record (black) is compared against the most aligned days with $I_{VEJ} > 95\%$ (red/gray).

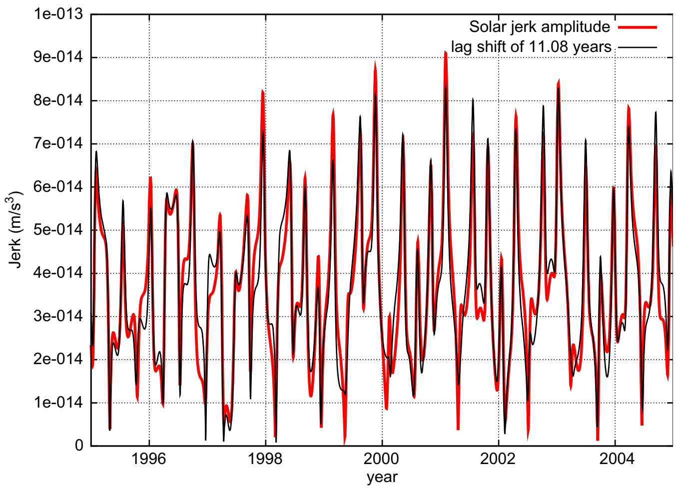

Fig. 4. Solar jerk amplitude depicted against itself with a time-lag of 11.08 years. Note the extremely good correlation that demonstrates the existence of a 11.08 year recurrence in the solar jerk function. The plotted periods are 1995–2005 (gray) and 2006.08–2016.08 (black), which is shifted back by 11.08 years.

The Sun's jerk vector is given by

$$\vec{J}_S(t) = \dot{\vec{a}}_S(t) = \sum_{P=1}^{8} \frac{Gm_p V_{SP}(t)}{R_{SP}^3(t)} \left(\frac{\vec{V}_{SP}(t)}{V_{SP}(t)} - 3\frac{\vec{R}_{SP}(t)}{R_{SP}(t)} \right), \tag{9}$$

where $\vec{V}_{SP}(t)$ is the velocity of the planet P relative to the Sun.

Eq. (9) is also closely related to the tidal acceleration induced by the planets, which is proportional to the factor $Gm_p/R_{SP}^3(t)$. The jerk vector weights the tidal acceleration with the velocity of each planet. If two planets induce the same tidal acceleration, the one that moves faster produces the greatest jerk vector on the Sun, and, thus, it induces the greatest jerk-stress and torsion on the solar plasma.

Fig. 4 shows the magnitude of the solar jerk vector given by Eq. (9). Fast fluctuations due to the movement of the inner planets are seen. The autocorrelation is maximum at 11.08-year lag. For example, the figure shows an extremely good correlation between the ten year period 1995–2005 and the ten year period 2006.08–2016.08.

Fig. 5 shows the angular evolution of the solar jerk latitude and longitude. The jerk vector rotates irregularly around the Sun following the planetary orbits. Latitude and longitude are plotted against themselves with a time-lag of 11.08-year, which again reveals a 11.08-year periodic cycle in the solar jerk vector.

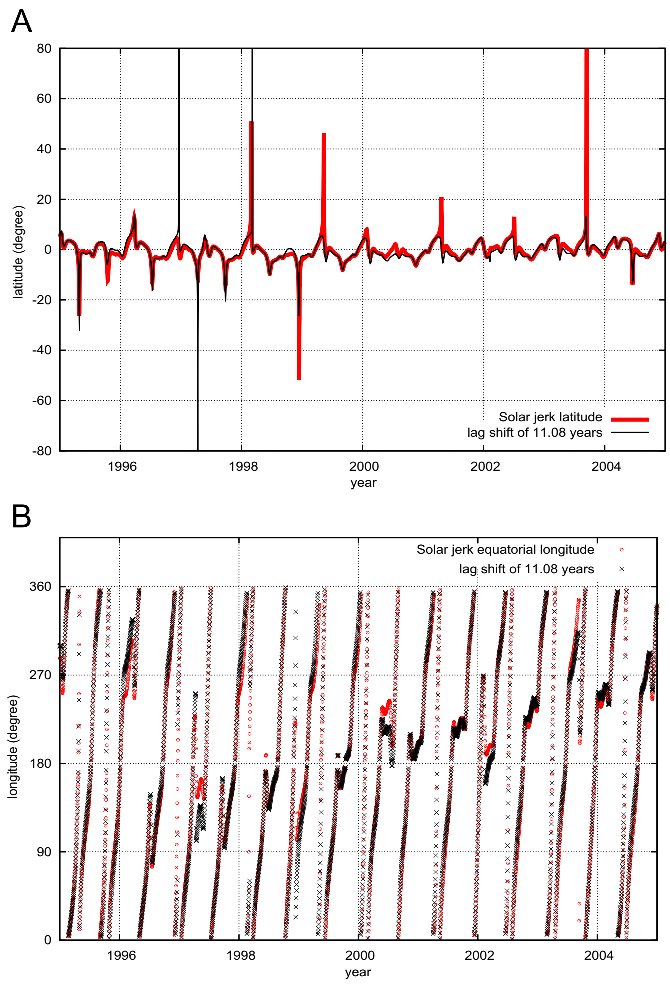

Fig. 5. (A) Solar jerk latitude and (B) longitude depicted against themselves with a time-lag of 11.08 years. Note the extremely good correlation that demonstrates the existence of a 11.08 year recurrence in the solar jerk vector rotation. The plotted periods are 1995–2005 and 2006.08–2016.08 as in Fig. 4.

Table 1
Amplitude of the jerk vector on the Sun for each planet: see Eq. (11).

Planet	Mass (kg)	Semi-major axis (m)	J_P (m/s^3)
Me	3.30E+023	5.79E+010	1.72E−14
Ve	4.87E+024	1.08E+011	2.84E−14
Ea	5.97E+024	1.50E+011	1.12E−14
Ma	6.42E+023	2.28E+011	2.76E−16
Ju	1.90E+027	7.79E+011	1.11E−14
Sa	5.69E+026	1.43E+012	3.92E−16
Ur	8.68E+025	2.88E+012	5.22E−18
Ne	1.02E+026	4.50E+012	1.28E−18

By assuming circular orbits and using the third Kepler's law, the speed of each planet, as seen from the Sun, is given by

$$V_{SP} = \sqrt{\frac{GM_S}{R_{SP}}}. \tag{10}$$

Thus, the amplitude of the solar jerk vectors corresponding to each planet is

$$J_P = \frac{G^{1.5} m_p \sqrt{10 M_S}}{R_{SP}^{3.5}}. \tag{11}$$

The vector is oriented on the planetary orbital plane at an angle

$$\beta = atan(-\tfrac{1}{3}) = +161.5651^\circ \tag{12}$$

from the Sun–planet distance vector. Table 1 shows the values of J_P for each planet. Venus and Mercury dominate this variable. Earth and Jupiter produce weaker and almost compatible jerk vectors.

Note that Mercury and Venus orbital combination repeats almost every 11.08 years. In fact, $46P_M = 11.086$ year and $18P_V = 11.07$ year, where P_M=0.241 year and P_V=0.615 year are the sidereal periods of Mercury and Venus, respectively. Indeed, Mercury and Venus's orbits repeat every 5.54 years, but the 11.08 year periodicity would better synchronize with the orbits of the Earth and with the tidal cycles of the sub-system Jupiter–Saturn. This resonance also implies that the tidal cycle produced by Mercury and Venus repeats every 5.54 and 11.08 years.

The 11.08 year recurrent pattern in the orbit of Mercury and Venus as well as the 11.07 year cycle in the alignment of Venus, Earth and Jupiter, which was discussed above (Eq. (2)), are particularly interesting because the mean solar cycle length since 1750 is estimated to be 11.06 years (in 1900 Brown found 11.1-years), which is extremely close to such planetary resonance periods. Perhaps these frequencies slightly shift the central Schawabe frequency from 10.81-year to 10.87-year (Scafetta, in press).

3. Estimation of the planetary tidal heating and its nuclear fusion amplification factor in the solar core

In this section we attempt to roughly estimate the luminosity variation that planetary tides can hypothetically induce, and we compare our estimate against the observed roughly 1 W/m^2 11-year TSI cycle variation at 1 AU, as seen in Fig. 1.

As explained in the Introduction, the idea that we propose is that nuclear fusion mechanisms in the solar core greatly amplify the gravitational energy released by tidal forcings. Only by means of a huge nuclear fusion amplification mechanism may planetary tides produce irradiance output oscillations with a sufficient magnitude relative to a background noise to influence solar dynamo processes. In fact, it is very unlikely that, dynamical mechanisms such as synchronization and resonance effects can be activated without an initial strong nuclear fusion amplification of the original gravitational tidal energetic signal, which by alone would be far too weak compared with competing energetic random flows present in the solar convective zone, as also indirectly argued in de Jager and Versteegh (2005). Here we explain how to calculate at first approximation a nuclear magnification factor converting released gravitational potential energy into solar luminosity by postulating the existence of a physical link between the solar luminosity rate and the potential energy associated to the mass lost by nuclear fusion in an unit of time. The needed equation can be deduced from the well-known *mass-luminosity relation* (Duric, 2004).

3.1. Basic theory of tides

The near-surface rotation rate of the Sun is observed to be larger at the equator (about 25 days) and to decrease as latitude increases (up to about 36 days at the poles) (Beck, 1999): current measurements are commonly expressed by a function of the form of $A+B\sin^2(\theta)+C\sin^4(\theta)$, where θ is the latitude of the Sun. The radiative zone of the Sun is found to rotate roughly uniformly with a period of about 26–27 days, while the core may rotate with a compatible rate (Thompson et al., 2003). Because herein we are only interested in finding a rough estimate of the effects of the tides on the Sun, for simplicity, we assume that the solar interior is rotating uniformly with a period of 26.5 days.

The basic tidal potential height equation is

$$H_t(\alpha,r)=\tfrac{3}{2}H_{tm}(r)[\cos^2(\alpha)-\tfrac{1}{3}], \quad (13)$$

where α is the angle, relative to the solar center, between the position of a planet and the position of a location A on the Sun, and $H_{tm}(r)$ is the high of the tide at $\alpha=0$ and distance r from the center of the Sun (Lamb, 1932). The value of $H_{tm}(r)$ depends on several variables such as the distance between the Sun and the planet, the distance r between the center of the Sun and the location A, and the mass of the Sun within the radius $r<R_S$.

Because the planetary tides are very small we assume the Sun to be perfectly spherical and use the average solar radius, $R_S=6.956\times10^8$ m. According to such approximation, the difference between the highest (at $\alpha=0$) and the lowest (at $\alpha=\pi/2$) tidal points, which is called *tidal elongation*, is given by (Taylor, 2005)

$$T_E(r)=\frac{3}{2}H_{tm}(r)=\frac{3}{2}\frac{m_P}{m_S(r)}\frac{r^4}{R_{SP}^3}, \quad (14)$$

where m_P is the mass of a planet P, $m_S(r)$ is the mass of the Sun included within the radius $r\le R_S$ and R_{SP} is the distance between the Sun and the planet P.

The initial factor 3/2 in Eq. (14) could be substituted with other *Love* numbers, which depend on the physical properties of the body deformed by the tides. The adoption of the Love factor 3/2, which refers to the tidal deformation of a small fluid shell above a rigid sphere, approximately like the case of the ocean tides above the Earth's more rigid crust, gives a lower limit. However, a Love factor 15/4 should be used if the perturbed Sun assumes the form of an uniform spheroid, which refers to an uniform fluid ball (Fitzpatrick, 2010). The Love number for the Sun may be between the two situations. In the following we use the factor 3/2 in the equations, but in the figures and in the tables we will use a double index referring to both Love numbers: 3/2 and 15/4. Note that the ratio between the two Love numbers is 2.5, so it is easy to convert one result into the other.

For example, if we use the Love number 3/2, given the solar total mass ($M_S=1.989\times10^{30}$ kg), on the Sun's surface Jupiter and Venus induce tidal elongations of about 0.68–0.71 mm, and Earth and Mercury induce tidal elongations of about 0.30–0.31 mm: see Table 2. The other planets induce even smaller tidal elongations. The highest combined tides are produced during the synodic/opposition periods among the planets: the *spring tides*. For example, a Jupiter–Venus spring tidal elongation is 1.4 mm on average. The minimum combined tides occur when right angles among the planets are formed: the *neap tides*. Table 2 also reports the difference of the tidal elongations calculated at the perihelion and at the aphelion of each planet. In this case Mercury produces the largest maximum orbital tidal variation (about 0.43 mm), Jupiter follows (about 0.21 mm), and Earth, Venus, Saturn and Mars have orbital tidal variation from 0.03 mm to 0.005 mm. These numbers are extremely small and would still be tiny if we adopt the Love number 15/4 and multiply them by 2.5: see Table 2. No planetary physical effect on the Sun would be measurable unless some huge internal amplification energetic mechanism exists. In fact, it is highly unlikely that internal dynamical synchronization and resonance mechanisms alone could be activated by such tiny tides because the gravitational tidal effect alone would be completely covered by dynamical noise in the convective zone, and the noise would efficiently disrupt the signal before it may become measurable. In fact, solar dynamics needs first to synchronize to the planetary frequencies, but if the thermal and dynamical noise is far too strong relative to the original tidal signal, numerous phase slips would occur, no synchronization would take place (Pikovsky et al., 2001) and resonance mechanisms would not be activated.

By comparison, the tidal elongations induced by the Moon ($R_{EM}=3.844\times10^8$ m and $M_M=7.348\times10^{22}$ kg) and by the Sun on the Earth ($R_E=6.37\times10^6$ m) are much larger: 0.54 m and 0.25 m, respectively, which are measurable quantities, and dynamical resonances may increase the signal up to a factor 100 in certain locations. Thus, we may expect that synchronization and dynamical resonance mechanisms alone could eventually increase the planetary tidal signal on the Sun by one or two orders of magnitude at most, but nothing would be observed if the energetic gap would be of six or more order of magnitudes.

Table 2
Average theoretical tidal elongation at the solar surface, due to the eight planets of the solar system (Eq. (14)). For the Sun we use $M_S=1.99$E30 kg and $R_S=6.96$E8 m. The 'diff tidal elongation' refers to the difference of the tidal elongations calculated at the perihelion and the aphelion, respectively. The last column reports the solar rotation as seen from each planet using a sidereal solar rotation of 26.5 days: longer values imply slower vertical tidal movement and less tidal work in unit of time. The tidal elongations are calculated with the Love numbers 3/2 and 15/4, the latter in parentheses.

Planet	Mass (kg)	Semi-major axis (m)	Perihelion (m)	Aphelion (m)	Tidal elong. (m)	Diff. tidal elong. (m)	S. rot. (days)
Me	3.30E23	5.79E10	4.60E10	6.98E10	3.0E−4 (7.5E−4)	4.3E−4 (1.1E−3)	37.92
Ve	4.87E24	1.08E11	1.08E11	1.09E11	6.8E−4 (1.7E−3)	2.6E−5 (6.6E−5)	30.04
Ea	5.97E24	1.50E11	1.47E11	1.52E11	3.2E−4 (7.9E−4)	3.2E−5 (7.9E−5)	28.57
Ma	6.42E23	2.28E11	2.07E11	2.49E11	9.6E−6 (2.4E−5)	5.5E−6 (1.4E−5)	27.56
Ju	1.90E27	7.79E11	7.41E11	8.17E11	7.1E−4 (1.8E−3)	2.1E−4 (5.2E−4)	26.66
Sa	5.69E26	1.43E12	1.35E12	1.51E12	3.4E−5 (8.5E−5)	1.2E−5 (2.9E−5)	26.57
Ur	8.68E25	2.88E12	2.75E12	3.00E12	6.4E−7 (1.6E−6)	1.7E−7 (4.3E−7)	26.52
Ne	1.02E26	4.50E12	4.45E12	4.55E12	2.0E−7 (5.0E−7)	1.3E−8 (3.3E−8)	26.51

Indeed, using Newtonian classical physics alone, the tidal elongations induced by the planets on the Sun appear to be extremely small and have discouraged researchers from believing that planetary gravity could influence solar activity. However, the physical quantity of interest is not the tidal elongation or accelerations but the solar nuclear fusion feedback to the tidal work released to the Sun that has the potentiality of greatly amplifying the gravitational energetic tidal signal. In the following subsections we roughly estimate the tidal power dissipated inside the Sun and the solar nuclear fusion positive feedback response to it.

3.2. *Estimation of the planetary tidal gravitational power dissipated in the Sun*

In the following we estimate the total maximum power that the planetary tides may hypothetically release to the Sun. At each location of the Sun with spherical coordinates (r,θ,ϕ), in a given time interval Δt, a given mass

$$dm(r,\theta,\phi) = \rho(r) r^2 \sin(\theta)\, dr\, d\theta\, d\phi, \tag{15}$$

is potentially vertically moved (up or down) by $|\Delta h(r,\theta,\phi)|$ by the tidal forces. The function $\rho(r)$ is the density of the Sun at the distance r (we assume a perfect spherical symmetry).

Herein we do not take into account the effects of the horizontal tides because the maximum energy involved in the process is determined by the vertical oscillation. However, the movement of the horizontal tides is also important because, together with the vertical tides, it can induce a mixing of solar masses that can carry fresh fuel to deeper levels (Wolff and Patrone, 2010) and increase nuclear fusion rate. The absolute value in the above equation is used because a tide, by working against the solar inertia to gravitational deformation, would heat the solar masses both when it rises and when it falls.

The total power related to the work associated to the vertical movement of the tide in a given location is given by

$$\Delta P(r,\theta,\phi) = \frac{\Delta W(r,\theta,\phi)}{\Delta t} = \frac{|\Delta h(r,\theta,\phi)| g(r) \rho(r) r^2 \sin(\theta)\, dr\, d\theta\, d\phi}{\Delta t}, \tag{16}$$

where $g(r)$ is the acceleration of gravity of the Sun at the distance r. The total power is given by a spherical integration on the entire Sun because tides affect also the inner part of the Sun, not just the solar surface or the tachocline. However, only a very small fraction of this power is dissipated in the star. Thus, we write

$$P = \frac{1}{Q} \int_{r=0}^{R_S} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \Delta P(r,\theta,\phi). \tag{17}$$

The parameter Q^{-1} is called *effective tidal dissipation function*, and it is defined as the energy lost during one complete cycle over the maximum energy stored in the tidal distortion: $Q^{-1} = (1/2\pi E_o) \oint (-dE/dt)\, dt$ (Goldreich and Soter, 1966; Jackson et al., 2008).

The exact determination of the solar Q factor goes beyond the purpose of this work, and for the Sun it is unknown. This factor is usually estimated indirectly through an analysis of the temporal evolution of orbital parameters in binary solar systems. In fact, it is known that the tidal torques transfer angular momentum and energy from the planet's rotation into the satellite's orbital revolution. In the case of giant planets such as Jupiter, it is possible to study the orbits of their moons, and it is found a value of $Q \approx 10^5$ (Goldreich and Soter, 1966). In the case of stars, the same methodology can be applied to the orbital evolution of binary systems (Tassoul, 2000), and for solar-like stars a reasonable range is $5 \times 10^5 \lessapprox Q \lessapprox 2 \times 10^6$, according to the empirical results depicted in Fig. 1 in Ogilvie and Lin (2007): see also Meibom and Mathieu (2005). However, a very large uncertainty exists about the parameter Q also because it is a function that depends on several variables such as the frequency and strength of the tides, and largely changes from star to star. Herein we adopt a constant value for all planetary tides:

$$Q = 10^6, \tag{18}$$

observing that there may be an error of one order of magnitude and that Q may be larger (smaller) for tides with larger (smaller) amplitude and frequencies: we will address these important corrections in another work. The large value of Q indicates that only a tiny fraction of the tidal associated power can be transferred to the Sun because of the solar interior rigidity.

The acceleration of gravity at a certain position r is given by

$$g(r) = \frac{G m_S(r)}{r^2}, \tag{19}$$

where the solar mass $m_S(r)$ within the radius $r \le R_S$ is given by

$$m_S(r) = 4\pi \int_0^r \rho(r') r'^2\, dr'. \tag{20}$$

Fig. 6 shows a typical solar density function $\rho(r)$ obtained from standard solar models (Bahcall et al., 2005).

In the following we consider the tide produced by only one planet P. The vertical displacement $|\Delta h(r,\theta,\phi)|$ can be calculated using Eq. (13). If the position of a planet P at a given time t in Cartesian coordinates is given by (X_t, Y_t, Z_t), and the position of a point A in the spinning Sun is (x_t, y_t, z_t) (in solar equatorial coordinate plane), we have

$$\cos(\alpha_{P,t}) = \frac{x_t X_t + y_t Y_t + z_t Z_t}{r R_{SP}(t)} = \frac{\sin(\theta)\cos(\phi_t) X_t + \sin(\theta)\sin(\phi_t) Y_t + \cos(\theta) Z_t}{R_{SP}(t)}, \tag{21}$$

where $R_{SP}(t)$ is the distance between the center of the Sun and the planet at the time t, and θ and ϕ_t are the latitude and the longitude of the solar location. The angle ϕ_t depends on t because of the solar rotation.

Here, for simplicity, we suppose that the Sun is rotating at a constant angular velocity $\Omega = 2\pi/T$, where T=26.5 days, as explained above. The vertical tidal displacement in a time interval Δt due to the planet P is calculated using Eqs. (13) and (14), and it

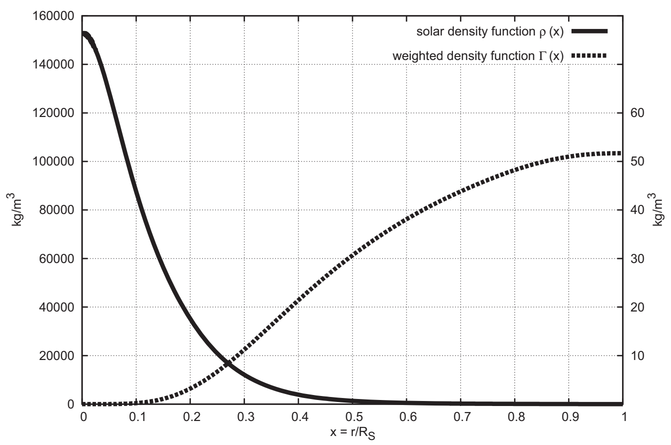

Fig. 6. A typical standard solar model density function $\rho(r/R_S)$ (solid line). The figure depicts the model BS05(OP) (Bahcall et al., 2001, 2005) which is constructed with a traditional understanding of the current heavy element abundance on the Sun. The weighted solar density function $\Gamma(r/R_S)$, Eq. (25) (dash line) refers to the values at the right ordinates. Three values may be of special interests: at the solar core radius, $\Gamma(0.3) = 11.23$ kg/m^3; at the solar radiative zone radius, $\Gamma(0.714) = 44.5$ kg/m^3; and at the solar near-surface radius, $\Gamma(1) = 51.7$ kg/m^3.

is given by

$$|\Delta h_P(r,\theta,\phi_t)| = \frac{3r^4 m_P}{2m_S(r)}\left|\frac{\cos^2(\alpha_{P,t})-\frac{1}{3}}{R^3_{SP}(t)} - \frac{\cos^2(\alpha_{P,t-\Delta t})-\frac{1}{3}}{R^3_{SP}(t-\Delta t)}\right|, \tag{22}$$

where the angles $\alpha_{P,t}$ and $\alpha_{P,t-\Delta t}$ are calculated using Eq. (21) by taking also into account the rotation of the Sun

$$\phi_{t-\Delta t} = \phi_t - \Omega\Delta t. \tag{23}$$

By substituting the above equations in Eq. (17), we find that the maximum tidal released power within the radius fraction r/R_S is given by

$$P_P(r/R_S,t) = \frac{3Gm_P R_S^5}{2Q\Delta t R_{SP}^3}\int_0^{r/R_S}\chi^4\rho(\chi)\,d\chi \times \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} R^3_{SP}\left|\frac{\cos^2(\alpha_{P,t})-\frac{1}{3}}{R^3_{SP}(t)} - \frac{\cos^2(\alpha_{P,t-\Delta t})-\frac{1}{3}}{R^3_{SP}(t-\Delta t)}\right|\sin(\theta)\,d\theta\,d\phi, \tag{24}$$

where R_{SP} is the semi-major axis of a given planet P while $R_{SP}(t)$ is the actual Sun–planet distance at the time t, the density function $\rho(\chi)$ is depicted in Fig. 6, and $G = 6.674\times10^{-11}$ N(m/kg)^2 is the Newton's universal gravitational constant.

Fig. 6 also depicts the weighted solar density function

$$\Gamma(r/R_S) = \int_0^{r/R_S}\chi^4\rho(\chi)\,d\chi. \tag{25}$$

Three values may be of special interests: at the solar core radius, $\Gamma(0.3) = 11.23$ kg/m^3, inside which solar fusion takes place; at the solar radiative zone radius, $\Gamma(0.7) = 43.96$ kg/m^3; and at the solar near surface radius, $\Gamma(1) = 51.7$ kg/m^3. The core and radiative zone radii are taken from Carroll and Ostlie (2007).

Note that a comparison between Eqs. (14) and (24) indicates that, while the tidal elongation is proportional to the fourth power of the solar radius, the power associated to the tidal movement is proportional to the fifth power of the solar radius. This property greatly increases the importance of tidal work versus tidal elongation. However, as shown in the next subsection, the released tidal gravitational power is at least one million times smaller than the TSI fluctuations, which indicates that Newtonian physics alone would rule out any observable planetary influence on the Sun.

3.3. Nuclear fusion feedback amplification and conversion of tidal gravitational released power into TSI at 1 AU

Herein we propose a theory based on modern physics. Every second nuclear fusion transforms a certain amount of mass into luminosity that is radiated away. With the vanished mass, also the gravitational energy of the star would change by a certain amount U_f. However, as each ionized atom of helium forms from four ionized atoms of hydrogen, the helium sinks toward the center and space in the core is freed; the pressure would decrease and the Sun would cool if the freed core space is not promptly filled, and the lost mass replaced by additional sinking hydrogen. This mass movement releases also a certain amount of gravitational energy, U_m, as thermal energy to the core. It is possible that to keep the core fusion activity sufficiently steady, as it is observed for hydrogen-burning main-sequence stars, this complex dynamics occurs in such a way to restore the previous gravitational energetic solar configuration: that is, we can postulate that: $U_f + U_m = 0$. Thus, there should exist a relation between solar luminosity and the gravitational power continuously dissipated in Sun by its gravity, which, in turn, should be closely related to the gravitational power associated to the mass vanished by nuclear fusion activity.

For example, the Kelvin–Helmholtz's contraction mechanism continuously causes a warming in the Jovian planets and in brown dwarfs. In fact Jupiter and Saturn emit almost twice as much energy as they receive from the Sun. The additional energy is believed to come from a gradual gravitational contraction and from the helium slow sinking that continuously convert gravitational potential energy into thermal energy (Carroll and Ostlie, 2007; Bennett et al., 2010). In the case of a planet the ratio between the gravitational released power and the emitted luminosity would be 1, if no amplification from nuclear fusion occurs. If nuclear fusion occurs, an appropriate amplification factor A would suffice to describe the phenomenon.

Planetary tides would do two things to the Sun: (1) they would add a little bit of dissipated gravitational power to that already released by the solar gravity itself, a fact that should slightly increase the average fusion rate in the core and (2) they add a small oscillation to the gravitational energy dissipated in the Sun because the tidal amplitude varies in time, which also forces the nuclear fusion rate to oscillate as well. An oscillating fusion rate would also force a small expansion/contraction oscillation of the core, which produces gravitation waves that propagate fast in the Sun from the core to the convective zone and promptly transport the energy signal, as it is better explained in the final paragraph of Section 5.

Point #1 above suggests the strategy that we need to adopt to solve the problem. In fact, the tidal work on the Sun would have essentially the same luminosity increase effect that a star like the Sun, without planets, would have if its mass is increased by an appropriate small amount ΔM. The Hertzsprung–Russell diagram for main sequence stars establishes that, if the mass of a star increases, its luminosity, L, increases as well according to the well-known *mass-luminosity relation*. In the case of a main-sequence star with mass $M = M_S + M$ close to our solar mass, M_S, the mass-luminosity relation is approximately given by

$$\frac{L}{L_S} \approx \left(\frac{M}{M_S}\right)^4 \approx 1 + \frac{4\Delta M}{M_S}, \tag{26}$$

where L_S is the luminosity of our Sun (Duric, 2004). Eq. (26) is good for stellar masses within the interval $0.43 < M/M_S < 2$. As explained above, we can assume that the Sun works as a system that every second transforms into luminosity the amount of gravitational energy associated to the transformed mass. Thus, we can postulate the existence of an equation similar to Eq. (26) and say that if we indicate with $\dot{U}_{tidal}$ the small gravitational power released by tidal work in the Sun and with $\dot{U}_{Sun}$ the average potential power released by solar gravity alone as a consequence of the nuclear fusion activity, the Sun works in such a way to also transform $\dot{U}_{tidal}$ into luminosity according the following relation

$$L(t) \approx L_S + 4L_S\frac{\dot{U}_{tidal}(t)}{\dot{U}_{Sun}} = L_S + A\cdot\dot{U}_{tidal}(t), \tag{27}$$

where A is the amplification factor and $L_{tidal}(t) = A\cdot\dot{U}_{tidal}(t)$ represents the small luminosity anomaly induced by the tidal work on the Sun, which oscillates in time with the orbits of the planets. To the first order approximation, the above equation can be used to convert tidal dissipated work into solar luminosity anomaly once that $\dot{U}_{Sun}$ is calculated. In the following we check our hypothesis by calculating the solar response to tidal dissipated energy as described by Eq. (27), and compare our result to the observed TSI fluctuations.

The total solar luminosity is

$$L_S = 4\pi(1\ \mathrm{AU})^2\times \mathrm{TSI} = 3.827\times10^{26}W, \tag{28}$$

where 1 AU $= 1.496\times10^{11}$ m is the average Sun–Earth distance,

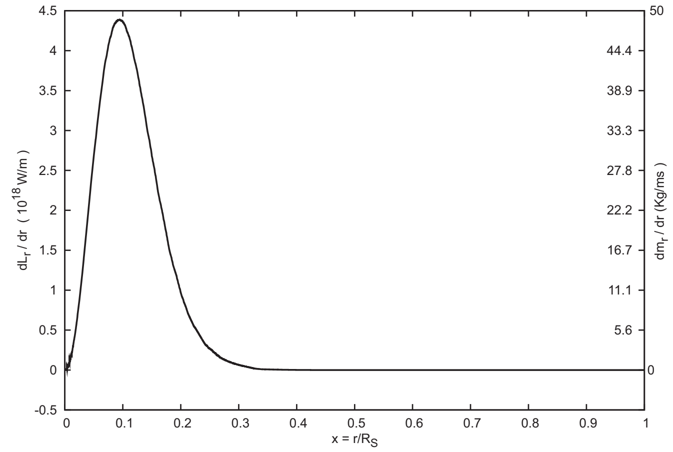

Fig. 7. Left units: derivative of the interior solar luminosity as a function of radius (Bahcall et al., 2001; Carroll and Ostlie, 2007). Right units: derivative of the interior solar burned mass in one second as a function of radius: see Eq. (29). A similar shape is also shown by the amplification conversion function $K(\chi)$, Eq. (32).

and TSI is the total solar irradiance, TSI = 1360.94 W/m^2, at 1 AU. This luminosity is produced mostly in the stellar core. The luminosity density function is depicted in Fig. 7 (Carroll and Ostlie, 2007). The solar luminosity is generated by nuclear fusion. The function of the interior mass transformed into energy in one second, $\dot{m}(r)$, is also indicated by the curve depicted in Fig. 7, and it is given by the well-known Einstein's equation ($E = mc^2$)

$$\frac{d\dot{m}(r)}{dr} = \frac{1}{c^2}\frac{dL(r)}{dr}, \tag{29}$$

where $c = 2.998 \times 10^8$ m/s is the speed of the light. Because nuclear fusion converts mass into energy that is radiated away, an equivalent amount of gravitational potential energy disappears as well. We assume that, at first approximation, this potential energy is compensated by additional gravitational work. Here we postulate that the rate of total solar gravitational released energy that we need to use is associated to the lost mass from Eq. (29), and given by

$$\dot{U}_{Sun} = \frac{1}{2} G \int_0^{R_S} m_S(r) \frac{d\dot{m}(r)}{dr}\frac{1}{r}\, dr = 3.6 \times 10^{20} W, \tag{30}$$

where the initial factor 1/2 is due to the virial theorem. Thus, using Eq. (27), a first order magnitude of the average amplification factor for converting gravitational potential released power into solar luminosity is

$$A = \frac{4L_S}{\dot{U}_{Sun}} \approx 4.25 \times 10^6. \tag{31}$$

The above amplification factor should be distributed inside the Sun according to the luminosity density function depicted in Fig. 7. Thus, the function for converting gravitational power into TSI at a distance $D = 1$ AU $= 1.496 \times 10^{11}$ m is

$$K(\chi) = \frac{A}{4\pi D^2}\frac{R_S}{L_S}\frac{dL(r)}{dr} = \frac{A}{4\pi D^2}\frac{1}{L_S}\frac{dL(\chi)}{d\chi}, \tag{32}$$

where $\chi = r/R_S$. Note that $K(\chi)$ has the same shape of the function depicted in Fig. 7, which stresses the importance of the solar core, and that $\int_0^1 K(\chi)\, d\chi = A/(4\pi D^2) \approx 1.6 \times 10^{-17}$ m^{-2}.

3.4. Numerical estimate of the planetary tidal induced TSI oscillations at 1 AU

We combine Eqs. (24) and (32), and then integrate the product function to obtain the total solar irradiance anomaly function associated to the planetary tide as they evolve in time, $I_P(t)$. Thus, we obtain

$$I_P(t) = \frac{3Gm_P R_S^5}{2Q\Delta t R_{SP}^3}\int_0^1 K(\chi)\chi^4 \rho(\chi)\, d\chi \times \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} R_{SP}^3 \left| \frac{\cos^2(\alpha_{P,t}) - \frac{1}{3}}{R_{SP}^3(t)} - \frac{\cos^2(\alpha_{P,t-\Delta t}) - \frac{1}{3}}{R_{SP}^3(t-\Delta t)} \right| \sin(\theta)\, d\theta\, d\phi. \tag{33}$$

In the following we use the high-precision JPL HORIZONS ephemerides system to determine the actual daily positions of the planets. The data are sampled at a daily scale: that is, we choose

$$\Delta t = 24 * 3600 = 86,400 \text{ s}. \tag{34}$$

Let us define

$$I_P = \frac{3Gm_P R_S^5}{2Q\Delta t R_{SP}^3}\int_0^1 K(\chi)\chi^4\rho(\chi)\, d\chi = 5.2 \times 10^7 \cdot \frac{m_p}{R_{SP}^3}. \tag{35}$$

The function $I_P(t)$ for each planet can be numerically determined using Eq. (33). The results are depicted in Figs. 8 and 9 and in Table 3. The observed oscillations are due to the elliptical orbits of the planets with the maximum occurring at the aphelion.

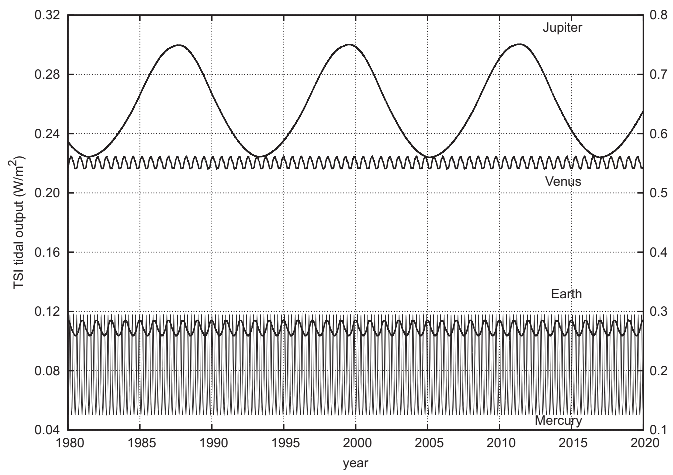

Fig. 8. Total tidal induced irradiance estimate $I_P(1,t)$ according to Eq. (33) for Jupiter, Venus, Earth and Mercury. The left scale refers to the Love number 3/2 and the right scale to 15/4.

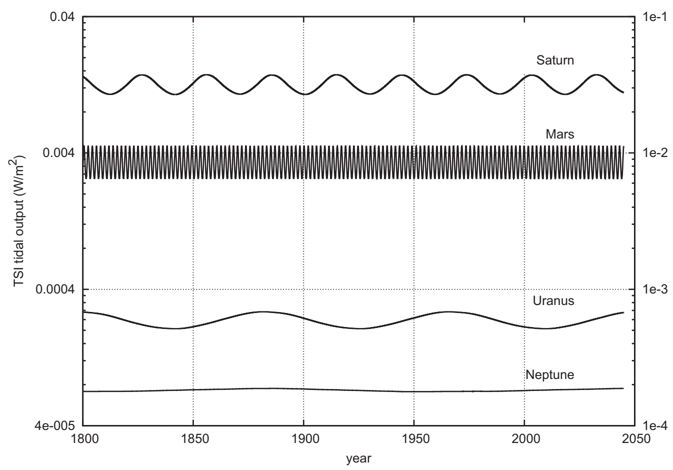

Fig. 9. Total tidal induced irradiance estimate $I_P(1,t)$ according to Eq. (33) for Saturn, Mars, Uranus and Neptune.The left scale refers to the Love number 3/2 and the right scale to 15/4.

Table 3
Values of I_P (Eq. (35)), and the average, max–min amplitude, min, the date of the max closest to 2000 (which is very close to the aphelion date), and period of the tidal irradiance estimate function $I_P(t)$ (Eq. (33)) for each planet. These curves are reported in Figs. 8 and 9. The values are calculated with the Love numbers 3/2 and 15/4, the latter in parentheses.

Planet	I_P (W/m^2)	Average (W/m^2)	Max–Min (W/m^2)	Min (W/m^2)	Max (W/m^2)	Max-date (year)	Period (year)
Me	8.8E−2 (2.2E−1)	8.4E−2 (2.1E−1)	6.8E−2 (1.7E−1)	5.0E−2 (1.3E−1)	1.2E−1 (3.0E−1)	2000.126	0.241
Ve	2.0E−1 (5.0E−1)	2.2E−1 (5.5E−1)	8.8E−3 (2.2E−2)	2.2E−1 (5.4E−1)	2.3E−1 (5.6E−1)	1999.918	0.615
Ea	9.3E−2 (2.3E−1)	1.1E−1 (2.7E−1)	1.0E−2 (2.6E−2)	1.0E−1 (2.6E−1)	1.1E−1 (2.9E−1)	2000.011	1.00
Ma	2.8E−3 (7.1E−3)	3.5E−3 (8.9E−3)	1.9E−3 (4.7E−3)	2.6E−3 (6.5E−3)	4.5E−3 (1.1E−2)	1999.893	1.88
Ju	2.1E−1 (5.2E−1)	2.6E−1 (6.6E−1)	7.6E−2 (1.9E−1)	2.2E−1 (5.6E−1)	3.0E−1 (7.5E−1)	1999.551	11.86
Sa	1.0E−2 (2.5E−2)	1.3E−2 (3.2E−2)	4.2E−3 (1.1E−2)	1.1E−2 (2.7E−2)	1.5E−2 (3.8E−2)	2003.258	29.46
Ur	1.9E−4 (4.8E−4)	2.4E−4 (6.0E−4)	6.8E−5 (1.7E−4)	2.1E−4 (5.2E−4)	2.7E−4 (6.9E−4)	1965.821	84.01
Ne	5.8E−5 (1.5E−4)	7.3E−5 (1.8E−4)	4.0E−6 (1.0E−5)	7.1E−5 (1.8E−4)	7.5E−5 (1.9E−4)	2051.43	164.8

Jupiter produces the largest effect both in average and in the amplitude of the oscillation. Venus, Earth and Mercury and Saturn follow in order. The average values I_P (see Table 3) vary from about 0.08 W/m^2 to 0.24 W/m^2 at least for Mercury, Venus, Earth and Jupiter. Mars, Uranus and Neptune produce smaller effects. All calculations need to be amplified by 2.5 if the Love number 15/4 is used, and we would get a range between 0.2 W/m^2 and 0.6 W/m^2 for the same planets: see Table 3, and Figs. 8 and 9. Note that a frequency/amplitude dependency of the factor Q would change the relative weight of the tidal signals but, as explained above, this complex issue will be addressed in future work.

4. Combination of all planetary tides and their ~10–11–12–61 year TSI induced oscillations

Herein, we assume that all tides are linearly superimposed and re-write Eq. (33) as

$$I_P(t) = \frac{3GR_S^5}{2Q\Delta t}\int_0^1 K(\chi)\chi^4\rho(\chi)\,d\chi \times \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\left|\sum_{P=1}^{8} m_P\frac{\cos^2(\alpha_{P,t})-\frac{1}{3}}{R_{SP}^3(t)} - m_P\frac{\cos^2(\alpha_{P,t-\Delta t})-\frac{1}{3}}{R_{SP}^3(t-\Delta t)}\right|\sin(\theta)\,d\theta\,d\phi, \quad (36)$$

where the internal sum is extended to all eight planets. Eq. (36) is numerically integrated, and the output is depicted in Fig. 10. The figure shows very rapid oscillations due to the inner planets. The average is about 0.35 W/m^2, which is about 250 ppm of the TSI output. The fast oscillations have a bottom-to-top amplitude up to about 0.65 W/m^2. Alternatively, if the Love number 15/4 is used, we would get 0.88 W/m^2, 1.63 W/m^2 and 625 ppm, respectively.

A simple 1001-day moving average reveals irregular cycles dominated by Jupiter's orbital period of 11.86 years with a bottom-to-top amplitude up to about 0.1 W/m^2 (or 0.25 W/m^2, alternatively). In addition, a beat oscillation of about 61 years is observed. This beat frequency reveals the presence of another frequency with a period of about 10 years, which is due to the spring tidal period of Jupiter and Saturn (see Eq. (1)). The figure shows that these minima occurred around 1910–1920 and 1970–1980, when solar minima as well as Earth's global surface temperature minima, are observed (Scafetta, 2009, 2010, 2012, in press).

Fig. 11 shows an integration of Eq. (36), where the planets are assumed to have circular orbits at their semi-major axis distance. The motivation is to study the tidal effect of the planetary orbital average. In fact, the solar internal dynamics may more easily dampen the oscillations due to the elliptical shape of the orbits, and, consequently, may respond more strongly to the tide as calculated at the average distance of the planets from the Sun, which is given by the semi-major axis. A simple 1001-day moving average of the integrated record reveals also irregular cycles, but now the cycles oscillate around a 10-year period with a max–min amplitude of about 0.05 W/m^2 (or 0.125 W/m^2, alternatively).

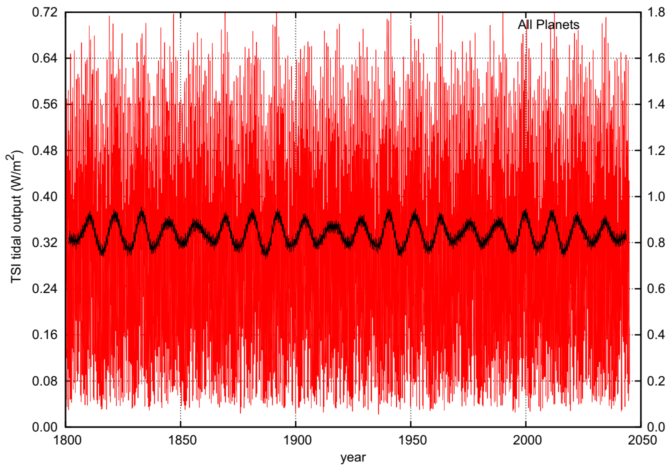

Fig. 10. Total tidal induced irradiance estimate according to Eq. (36) obtained with a linear local superposition of all planetary tides. A 1001 day moving average curve is added and reveals an approximate 12 year cycle plus a 61-year beat cycle. The left scale refers to the Love number 3/2 and the right scale to 15/4.

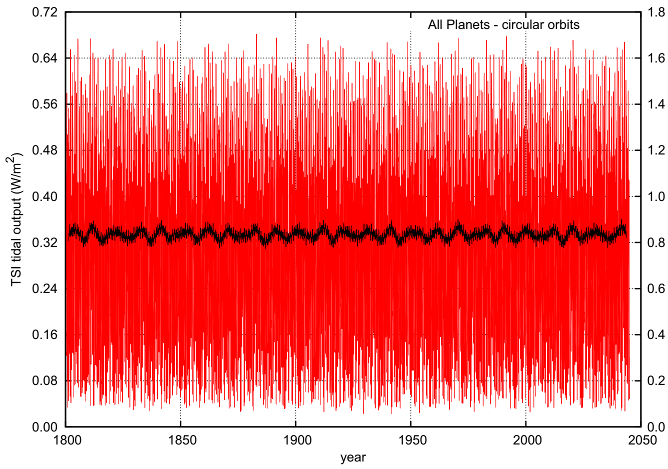

Fig. 11. Like Fig. 10 but now the total tidal induced irradiance is calculated by keeping the planets at their semi-major axis (sma). A 1001 day moving average curve is added and reveals an approximate 10 year cycle. The left scale refers to the Love number 3/2 and the right scale to 15/4.

Finally, Fig. 12 depicts the tidal effects of two separated planetary subsystems: (a) Jupiter and Saturn and (b) Mercury, Venus and Earth. Fig. 12A highlights the existence of the ~10-year Jupiter–Saturn spring tide cycle and the ~12-year Jupiter

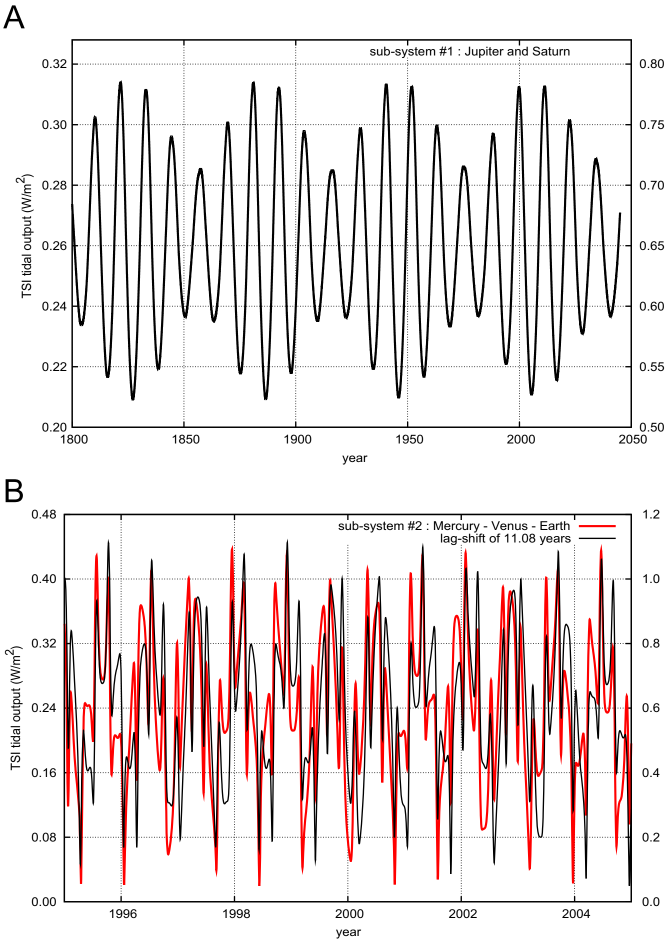

Fig. 12. Total tidal induced irradiance estimate according to Eq. (36). (A) the sub-system Jupiter and Saturn alone: note the ~10-year (Jupiter–Saturn spring tide) and ~12-year (Jupiter orbit) cycles that give origin to a ~61-year beat cycle (see Eq. (1)). (B) The sub-system Mercury, Venus and Earth; the periods 1995–2005 and 2006.08–2016.08 are superimposed to show an 11.08 year quasi-periodic recurrence in this tidal pattern. The left scale refers to the Love number 3/2 and the right scale to 15/4.

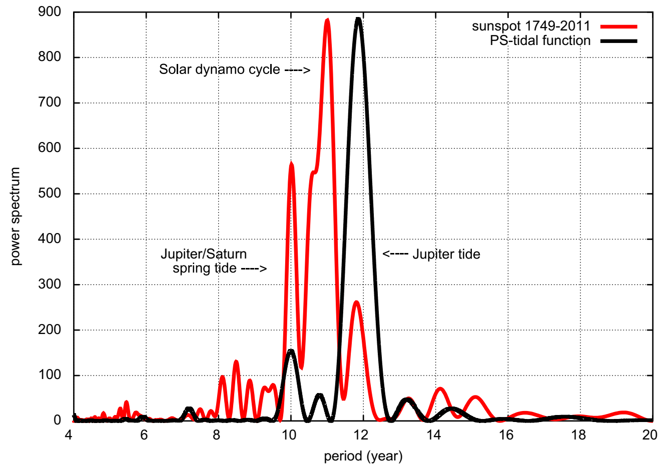

Fig. 13. Lomb-periodogram spectral analysis of the sunspot number record and of the total tidal function record depicted in Fig. 10. Note that the two side frequencies at about 10-year (J/S-spring tide) and 11.86-year (J-tide) correspond perfectly in the two spectral curves. The central frequency at about 10.9-year present in the sunspot number spectrum is likely generated by the solar dynamo itself while synchronizing its dynamics to the planetary frequencies. Note that the discrepancy in the relative amplitude of the side tidal peaks may be due to an internal physical mechanism that dampens one frequency relative to the other.

orbit cycle that generate also a ~61-year beat cycle (Eq. (1)). The first two cycles are clearly seen in the sunspot number record (see Fig. 2), and the ~61-year cycle is found in millennial cosmogenic isotope records such as ^{14}C and ^{10}Be, in the aurora records (Ogurtsov et al., 2002; Charvátová et al., 1988; Komm et al., 2003), and in numerous terrestrial climate records (Scafetta, 2010). Fig. 12B highlights the existence of the 11.08-year recurrent cycle generated by the terrestrial planets that well agrees with the 11.06-year mean sunspot number cycle length. The average tidal induced TSI is almost the same for the above two planetary subsystems: about 0.25 W/m^2 (or 0.625 W/m^2, alternatively). The induced oscillations in the two cases have amplitude of about 0.1 W/m^2 and 0.4 W/m^2 (or 0.25 W/m^2 and 1.0 W/m^2, alternatively).

The observed oscillations in Figs. 10 and 12 are compatible with the observed TSI oscillations ($\lesssim 1\ W/m^2$). Thus, the tidal induced fluctuations can be considered sufficiently energetic to activate synchronization and resonance processes yielding a further adjustment of the signal and/or the emergence of an additional solar cycle dynamo around 11 years (Scafetta, in press). However, as also explained above about the factor Q (Eq. 18), fast and/or large fluctuations are expected to be attenuated because of internal damping mechanisms, which are frequency dependent. These dynamic corrections will be addressed in future work.

Finally, in Fig. 13 we compare the Lomb-periodogram spectral analysis of the sunspot number record and of the total tidal function record depicted in Fig. 10. The figure focuses only on the frequency range of the Schwabe cycle. The Lomb-periodogram of the sunspot record confirms the finding of MEM depicted in Fig. 2 by showing three peaks at about 10, 11 and 11.86 years. As expected, the two side frequencies correspond to the frequencies of Jupiter/Saturn spring tide (at about 10 year period) and of Jupiter orbital tide (at about 11.86). Instead, the central frequency is likely generated by the solar dynamo which would tend to synchronize to the average beat tidal frequency as well as at the dynamical recurrence frequency (at 11.08 years) of the fast tides as shown in Fig. 12A. Scafetta (in press) showed that the three cycles beat producing known solar variability at multiple scales. Again, the frequency/amplitude dependency of the factor Q and of other internal synchronization and resonance mechanisms may explain why in the observation the ~11.86-year Jupiter cycle is damped relative to the 9.93-year Jupiter/Saturn spring tidal signal. Herein, we do not deal with these internal dynamical resonance mechanisms because we are only interested in determining whether the energy argument is sufficiently valid to further develop the theory in the future.

5. Rebutting possible objections

As explained in the Introduction, at least three major objections exist against a planetary influence on the Sun, and we believe that they can be rebutted.

The first objection (Smythe and Eddy, 1977) claims that Jupiter and Saturn planetary tidal patterns would be inconsistent with the periods of prolonged solar minima such as the Maunder grand solar minimum. This objection is explicitly addressed and rebutted in Scafetta (in press). There it is shown how the three frequencies detected in Fig. 2 and the tidal planetary timings can be used to develop an harmonic model capable of reconstructing all known major solar patterns. These include the correct timing of the 11-year Schwabe solar cycle and of the known prolonged periods of low solar activity during the last millennium such as the Oort, Wolf, Spörer, Maunder and Dalton minima, plus the emergence and the timing of the observed quasi-millennial solar cycle and others.

The second objection is based on Newtonian classical physics alone and claims that the planetary tides are too small to induce observable effects (de Jager and Versteegh, 2005, Callebaut et al., 2012). This paper rebuts this objection showing that there is the need to take into account modern physics and consider a nuclear fusion feedback response to tidal forcing that we have calculated to be able to amplify the gravitational tidal heating effect alone by more than a million time factor, see Eq. (31). Moreover, the tides act everywhere inside the Sun, not just at one level such as at the surface or at the tachocline. This implies that an integration of the tidal effect from the center to the surface is needed. Consequently, the total tidal effect on the Sun is proportional to R_S^5 as Eq. (33) shows, and not to R_S^4 as normally miscalculated by some critics who calculate the tidal elongation only at a predetermined distance from the solar center by simply using Eq. (14). Essentially, the Sun, by means of its nuclear active core, should work as a great amplifier that increases the strength of the small gravitational perturbation of the planetary tidal signals. We have found that planetary tides may induce an oscillating luminosity increase from 0.05–0.65 W/m^2 to 0.25–1.63 W/m^2. This signal should be sufficiently energetic to synchronize solar dynamics with the planetary frequencies and activate internal dynamical resonance mechanisms, which then generate and interfere with the solar dynamo cycle to produce the observed solar variability, as further explained in Scafetta (in press).

The third objection is based on the Kelvin–Helmholtz time scale (Mitalas and Sills, 1992; Stix, 2003) that would predict that the travel time scale of an erratic photon in hot plasma from the core to the convective zone ranges between 10^4 and 10^8 years. This argument is used to claim that even if the solar core gets warmer because of a tidal massaging, the luminosity perturbation would reach the surface on average after hundred thousand years. This time scale is very long compared to the historical astronomical record, and relatively small core luminosity variations would be practically smoothed out and disappear during the very long erratic photon transport journey to the surface. This topic is not directly addressed in the present paper because this paper focuses on the tidal heating effect in the solar core, not on how the energy may be transported to the surface.

Preliminary attempts to solve the above problem have been already proposed in the scientific literature, where it is assumed that the solar core is not in a perfect hydrostatic equilibrium because of the tidal heating. For example, Grandpierre (1990, 1996) proposed that planetary tides induce finite amplitude flows in the core that induce an electric field generation, which then produces some kind of gently local thermonuclear runaways which shoot up convective cells to the outer layers. Thermonuclear runaways processes move energy very fast at a speed of several kilometers per second, and are well known to cause supernova explosions. More recently, Wolff and Patrone (2010) argued that: "*an event deep in the Sun that affects the nuclear burning rate will change the amount of energy going into the g-mode oscillations. Some information of this is transported rather promptly by g-modes to the base of the Sun's convective envelope (CE). Once these waves deposit energy there, it is carried to the surface in a few months by extra convection, which should increase solar activity in the way described early in Section 1. This upward transport of luminosity by waves was also advocated by Wolff and Mayr (2004) to explain the east–west reversing flows detected by Howe et al. (2000) and Komm et al. (2003) with characteristic time scales of one to three years*". Indeed, if the solar nuclear fusion rate oscillates because of an oscillating planetary tidal forcing, it should cause gravitational perturbations through buoyancy waves that should be felt by the entire Sun quite fast. Thus, it is possible that the energy output variation due to tidal forcing would propagate through the solar interior by means of pressure waves at a very high speed. An imperfect analogy would be given by sound waves and by the pressure propagation in a fluid, which is regulated by Pascal Principle that states that the pressure applied to an enclosed fluid is transmitted undiminished to every part of the fluid and that the perturbation propagates with the speed of the sound in that specific fluid. If the core warms a little bit, it expands, and all solar interior should rapidly feel the associated gravitational/pressure effects. These pressure perturbations together with the increased core luminosity may have the effect of modulating the luminosity flux toward the tachocline and induce a harmonic modulated forcing of the convective zone that produces the final TSI output. Consequently, the luminosity output could respond fast to oscillating changes in fusion rate occurring in the core without the need to wait hundred thousand years so that the surplus photons produced in the core come out of the radiative zone.

It is through these wave pressure perturbations that the energy signal may be transferred from the core to the tachocline quite fast. For example, the internal solar g-wave oscillations could evolve as: $G(t)=(1+a\cos(\omega_p t))\cos\{[1+b\cos(\omega_p t)]\omega_g t\}$, where ω_g is the average frequency of the g-waves, ω_p is the planetary induced harmonic modulation, and a and b are two small parameters. Once at the tachocline, this energy anomaly would act as a modulating forcing of the solar dynamo and would tend to synchronize it (González-Miranda, 2004; Pikovsky et al., 2001; Scafetta, 2010) to its own frequencies generating a complex Schwabe cycle among other oscillations, as discussed above and in Scafetta (in press).

Finally, there are also some conflicting studies investigating whether stellar activity could be strongly enhanced by closely orbiting giant planets (Scharf, 2010; Poppenhaeger and Schmitt, 2011). However, until a sufficiently complete planetary–star interaction theory is developed, the issue cannot be conclusively solved by simply investigating other solar systems. In fact, we neither have long enough records nor detailed information about other stars and their solar systems. Moreover, poorly-understood interaction mechanisms, non-linear effects and stellar inertia mechanisms to fast and large tidal deformations may lead misleading conclusions. Evidently, a star–planet interaction theory can only be developed and tested by studying our Sun and our solar system first, as done herein and in Scafetta (in press).

6. Conclusion

Numerous empirical evidences indicate that planetary tides can influence solar dynamics. High resolution power spectrum analysis reveals that the sunspot number record presents three frequencies at about Jupiter/Saturn's spring tidal period of 9.93 years, at 10.87 ± 0.1 and at Jupiter period 11.86 years. In addition, the alignment patterns of the sub-systems of Venus–Earth–Jupiter and Mercury–Venus produce major resonance cycles at about 11.05–11.10 years, which coincides with the average length of the observed Schwabe sunspot cycles since 1750. Thus, the Schwabe solar cycle is reasonably compatible with the tidal cycles produced by the five major tidal planets: Mercury, Venus, Earth, Jupiter and Saturn. More details are found in Scafetta (in press), where it is shown how to reconstruct solar dynamics at multiple time scales using some of these frequencies.

Despite numerous empirical results, a planetary-solar link theory has been found problematic in the past mostly because the gravitational tides induced by the planets on the Sun are tiny, as deduced from the tidal Eq. (14). The major tidal planets (Mercury, Venus, Earth and Jupiter) would produce tides of the order of a millimeter due to the fact that the tidal elongation is proportional to R_S^4: see Eq. (14). However, it is the tidal work on the Sun that physically matters, and we have shown that the total work that the planetary tides may release to the Sun is proportional to R_S^5. Indeed, the tides should move up and down the entire column of solar mass. The tidal movement consistently and continuously squeezes and stretches the entire Sun from the center to the surface. The solar mass can be moved and mixed by gravitational tidal forces also because of the fluid nature of the solar plasma. However, even in this case only a tiny fraction of the gravitational tidal energy can be released as heat to the Sun (see Eq. (18)), and nothing would be expected to happen if only released tidal gravitational energy is involved in the process, as Newtonian classical physics would predict.

However, a planetary tidal *massaging* of the solar core should continuously release additional heat to it and also favor plasma fuel mixing. Consequently, the Sun's nuclear fusion rate should be slightly increased by tidal work and should oscillate with the tidal oscillations. In Section 3.3 we have proposed a methodology to evaluate a nuclear amplification function (Eq. (32)) to convert the gravitational potential power released in the core by tidal work into solar luminosity. The strategy is based on the fact that nuclear fusion inside a solar core is kept active by gravitational forces that continuously compress the core and very slowly release additional gravitational energy to it, as the hydrogen fuses into helium. Without gravitational work, no fusion activity would occur either because the two phenomena are strongly coupled (Carroll and Ostlie, 2007). Thus, a simple conversion factor should exist between released tidal gravitational power and its induced solar luminosity anomaly. We can estimate it using a simple adaptation of the well-known mass-luminosity relation for main-sequence stars similar to the Sun: see Eq. (27). The average estimated amplification factor is $A \approx 4.25 \times 10^6$, but it may vary within one order of magnitude. In fact, there is uncertainty about the Love number that in the case of the Sun may be larger than the used factor 3/2 (see Eq. (14)), and the effective tidal dissipation factor Q likely varies with the tidal frequency and amplitude, and may be different from the used binary-star average value $Q = 10^6$ (see Eq. (18)).

With the theoretical methodology proposed in Section 3.3 we have found that planetary tides can theoretically induce luminosity oscillations that are within one order of magnitude compatible with the TSI records. We have found that planetary tides may induce an oscillating luminosity increase from 0.05–0.65 W/m^2 to 0.25–1.63 W/m^2. Additional synchronization and resonance processes may be activated and produce an additional dynamical amplification effect. Internal frequency-dependent damping mechanisms may also be present. The solar dynamo cycle would also contribute to the final solar cycle as explained in Scafetta (in press). Although these internal dynamic processes are not addressed in this work, planetary tides appear to be able to modulate solar activity in a measurable way and our results are consistent with the observations.

Finally, we have shown that the planetary tides produce major cycles with 10, 11, 12 and 61 year periods, which correspond to the cycles observed in the sunspot number record and other solar and climate records (Ogurtsov et al., 2002; Charvátová et al., 1988; Komm et al., 2003; Scafetta, 2010). The cycles with periods of 10, 12 and 61 years are directly related to Jupiter and Saturn orbits; the 11-year cycle is the average between the 10 and 12 year Jupiter–Saturn cycles, and it is also well reproduced by the recurrent tidal patterns generated by the fast tidal cycles related to Mercury, Venus and Earth. The tidal heating generated by the two planetary subsystems (terrestrial and Jovian planets) is almost the same. So, terrestrial and Jovian planets should be both important to determine solar dynamics at multiple time scales. In particular we note from Fig. 12A that the combined tides of Jupiter and Saturn would imply an increased solar activity occurring from 1970 to 2000 with a peak around 2000 that would also be almost in phase with the 10.87-year solar dynamo cycle (Scafetta, in press): this pattern would be qualitatively consistent with the pattern shown by the ACRIM total solar irradiance composite depicted in Fig. 1 (Scafetta and Willson, 2009).

The preliminary results of this paper suggest that for better understanding solar activity, the physical interaction between the planets and the Sun cannot be dismissed, as done until now. Future research should better address the nature of these couplings, which could also be used to better forecast solar activity and climate change (Scafetta, 2010, in press). In fact, planetary dynamics can be rigorously predicted.

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